4. Theory

From measuring the circular trace diameter in a magnetic field, the electron charge-to-mass ratio (*e/m*) can be calculated with a formula which is derived as follows.

1) Setup & assumptions
   - Electrons are accelerated through a potential difference V (starting from rest).
   - They enter a region of uniform magnetic field B perpendicular to their velocity, so they move in a circular path of radius r.
   - Charge magnitude is e, mass m, speed after acceleration is v.

2) Energy → speed

   Energy gained by electron from the accelerating potential:

   $$eV = \tfrac{1}{2}mv^2$$

   Solve for v^2:

   $$v^2 = \frac{2eV}{m}.$$

3) Magnetic force → circular motion

   Magnetic Lorentz force provides centripetal force (for $v \perp B$):

   $$evB = \frac{mv^2}{r}.$$

   (Left: magnetic force magnitude evB; right: centripetal mv^2/r.)

   Cancel one v (assuming $v \neq 0$):

   $$eB = \frac{mv}{r} \quad \Rightarrow \quad v = \frac{eBr}{m}.$$

4) Combine the above two relations

   Substitute v from the magnetic relation into the energy equation:

   $$eV = \tfrac{1}{2}m\left(\frac{e^2B^2r^2}{m^2}\right) = \frac{e^2B^2r^2}{2m}.$$

   Rearrange to solve for e/m:

   $$2meV = e^2B^2r^2 \quad \Rightarrow \quad \frac{e}{m} = \frac{2V}{B^2r^2}.$$

5) Use the measured diameter D of the electron's trace in the magnetic field to calculate e/m

If you measure the trace diameter D on the screen, $r=D/2$. Substitute:

$$\frac{e}{m} = \frac{2V}{B^2(D/2)^2} = \frac{2V}{B^2D^2/4} = \frac{8V}{B^2D^2}.$$

where B is generated by using Helmholtz coils, the on-axis field (ideal Helmholtz spacing) is

$$B = \frac{\mu_0\, N\, I}{R}\left(\frac{4}{5}\right)^{3/2},$$

where N=turns per coil, I=coil current, R=coil radius, and $\mu_0=4\pi\times10^{-7}\ H/m$.

According to the specifications of this specific apparatus, we have $N=140$ turns, $R=28/2=14\ cm$, use them to simplify the calculation formula and the charge to mass ratio of an electron (e/m) can be calculated as:

$$\frac{e}{m} = 9.88\times10^6\,\frac{V}{I^2D^2}\ \text{(C/kg)},$$

where V and I are the acceleration voltage and the excitation current, respectively.

Since the cooper wire in the actual coil does not fully fill the coil frame, the diameters of the inner layer and the outer layer are respectively 26.4 cm and 28.8 cm, so the average diameter of the coil is 27.6 cm. Using this value, better calculation accuracy can be achieved, it is:

$$\frac{e}{m} = 9.61\times10^6\,\frac{V}{I^2D^2}\ \text{(C/kg)}.$$

5. Operational Instructions

1) Pre-experiment preparation

Read the instruction manual carefully before operating the apparatus.

The Lorentz force tube is pre-installed on the tube base with shockproof materials around it. <u>For first-time use</u>:

a) Open the front cover.

b) Remove the ruler from the coil frames.

c) Carefully take out all shockproof materials by holding the lower portion (i.e. the plastic part of the bulb neck) of the tube with one hand while removing the materials with the other.
d) Pay special attention to taking out the volt-ammeter.
e) Reinstall the ruler securely onto the coil frames.

Next, loosen the pin screw on the tube base and raise it out of the pin hole on the case top. This will allow the base with the tube to rotate freely (do not remove the screw completely from the base). After completing the experiment, resume the pin screw into the pin hole to secure the tube in place for protection.

Note: When installing or removing the Lorentz force tube, hold and apply force only to the **plastic part at the bottom of the glass bulb**. Do **not** grip or apply force to the glass portion, as this may cause the glass and plastic parts to loosen or separate.

The settings of the front panel should be as follows:

Knob for adjusting acceleration voltage: turned all the way to end of counterclockwise

Current polarity switch: off

Knob for adjusting excitation current: turned all the way to end of counterclockwise

Voltage polarity switch: off

Knob for adjusting deflection voltage: turned all the way to end of counterclockwise

Plug in the power cord and turn the power on to the Lorentz force demonstrator. The power indicator should be on. After a short period of warm-up (e.g. 3-5 minutes), the apparatus is ready for use.

2) Observe the moving trace of electrons in a magnetic field

a) Turn the adjusting knob for acceleration voltage clockwise to increase the voltage while monitoring a rectilinear trace of electrons in the absence of a magnetic field.

b) Revolve the Lorentz force tube to make the angular pointer point at approx. 90° and set the current polarity switch to clockwise. Turn the adjusting knob for excitation current clockwise to increase the current while monitoring the trace of electrons to start to incline (rotate the tube if necessary to ensure the trace of electrons parallel to the Helmholtz coils). The trace inclination of moving electrons is increased with an increase in the excitation current that is set at clockwise polarity. Eventually, a circular trace of moving electrons will be observed as shown in Figure 2.

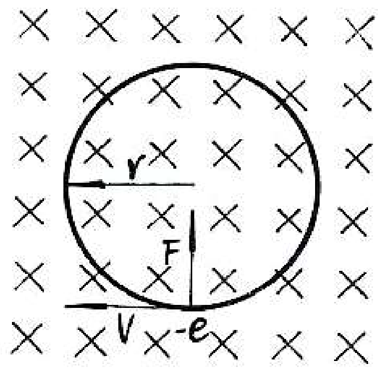


Figure 2 Trace of moving electrons in a magnetic field

The diameter of the circular trace is linearly proportional to the velocity of the moving electrons, but inversely proportional to the strength of the magnetic field. The circularity of the moving electrons increases with an increase in acceleration voltage but decreases with an increase in excitation current.

c) Revolve the Lorentz force tube to make the angular pointer point at 130° to 150° while monitoring the spiral trajectory of moving electrons. Revolve the Lorentz force tube to make the angular pointer point at 180°, the moving direction of electrons is now parallel to the direction of the magnetic field and therefore the magnetic field has no effect on the moving electrons. Thus, the trace of moving electrons becomes a rectilinear path.

Warning:

A. *When the instrument is idle, please reduce the acceleration voltage and the deflection voltage to minimum, also set the excitation current to minimum with the current polarity switch to "off", to extend the lifetime of the tube. When applying excitation current, conduct measurement as quickly as possible and then reduce the current to minimum.*

B. *Do **not rotate** the demonstrator tube more than **180° to either side**. The tube's base* *has delicate wire bonds underneath, and over-rotation beyond the 180° limit may twist and damage the wire bonds*.

3) Determine the charge to mass ratio of an electron

   Revolve the Lorentz force tube to make the angular pointer point at 90°. Set the acceleration voltage between 100 V to 200 V and the excitation current between 1.0 A and 2.0 A. Measure the diameter, D, of the electron beam circularity (viewing parallel to the side surface and top edge of the sliding pointer), as seen in Figure 3.

   The charge to mass ratio of an electron can be calculated as:

$$\frac{e}{m} = 9.61 \times 10^{6} \frac{V}{I^2 D^2} \text{ (C/kg)} \quad (1)$$

   where V and I are the acceleration voltage and the excitation current, respectively.

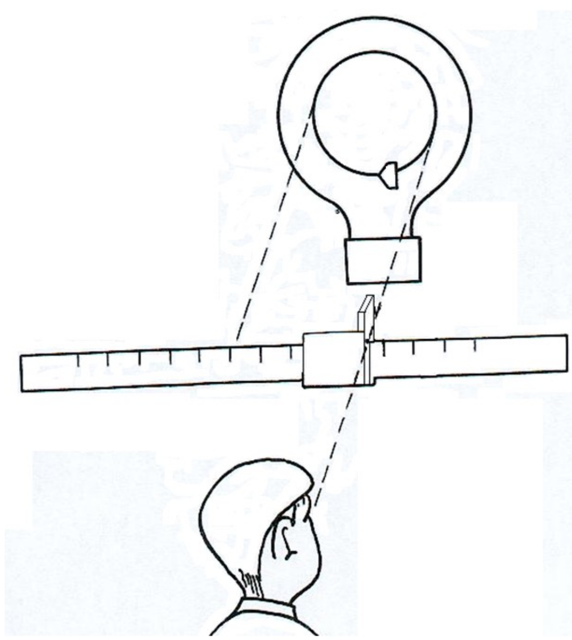
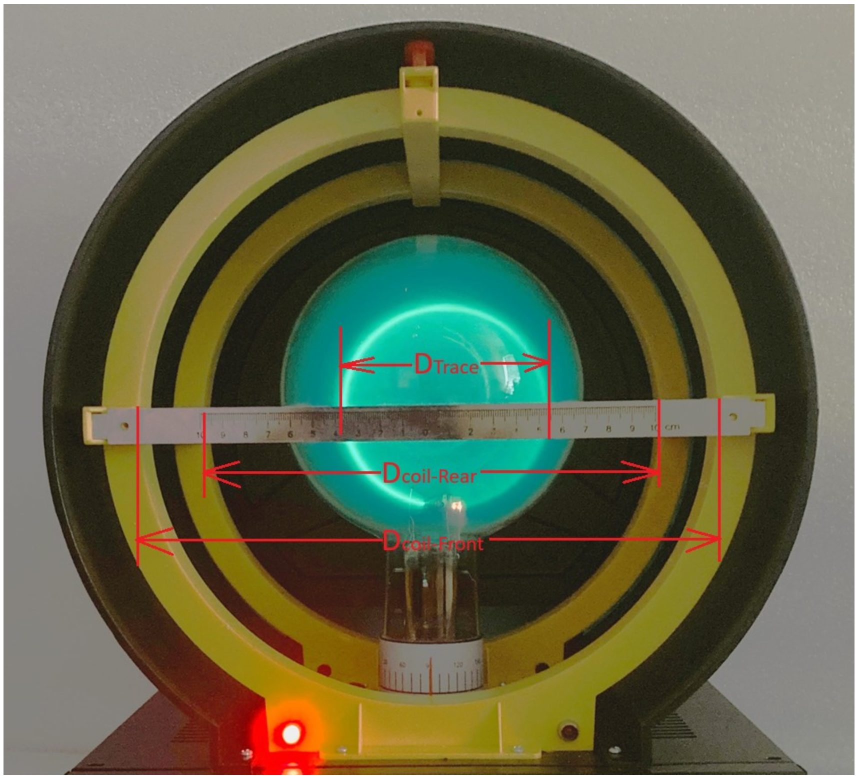


Figure 3 Schematic of measuring the diameter of electron beam trace

Method for improving the measurement accuracy of electron trace diameter:

Since the measuring ruler and the electron trace are not in the same plane, parallax occurs when measuring the diameter of the circular electron trace, leading to systematic error. To reduce this measurement error, we propose the following novel method:

a) Use a mobile phone to take a photo from a distance about 0.5m that clearly includes both the Helmholtz coils and the circular electron trace (see illustration below). Note: when framing the photo, align the camera's central axis as closely as possible with the common axis of the two coils. In other words, the resulting image should show the two coils as two concentric circles.



b) Enlarge the photo on a screen. Using a ruler (or by counting pixels), measure the apparent diameters of the front coil, the rear coil, and the electron trace, denoted as: $D_{coil\text{-}Front}$, $D_{coil\text{-}Rear}$ and D_{Trace}, respectively.

c) Assuming the electron trace lies exactly in the mid-plane between the two coils, a simple geometric derivation (left to the students to work out) gives the actual trace diameter D (unit is cm):

$$D = D_{\text{Trace}} \times \frac{26}{\left(\frac{D_{\text{coil-Front}} + D_{\text{coil-Rear}}}{2}\right)} \times 1.11$$
,

where 26 cm is the known inner diameter of the plastic frame of each Helmholtz coil, 1.11 is a correction factor for compensating the image off-axis distortion when the photo is taken at a distance of 0.5m (the value of correction factor depends on the distance, for example, it will be 1.024 when distance is 1.0m).

This acquired *D* value will provide a higher accurate measurement of the electron trace diameter compared with eye viewing.

The data in the table below are some examples of experiment results. They are for reference purposes only, not the criteria for apparatus performance.

Acc. Volt. (V)	Exc. Current (A)	$D_{coil-Front}$ (pixels)	$D_{coil-Rear}$ (pixels)	D_{Trace} (pixels)	Actual D (cm)	e/m ($\times 10^{11}$ (C/kg)
101	1.01	1496	1204	335	7.16	1.855
101	1.50	1493	1197	229	4.91	1.787
150	1.01	1487	1182	414	8.96	1.760
150	1.51	1485	1179	275	5.95	1.787
200	1.00	1502	1208	323	6.88	1.805
200	1.50	1490	1191	485	10.45	1.759
227	1.00	1500	1207	344	7.34	1.755
224	1.51	1493	1187	529	11.40	1.678
Measured e/m (average)						1.773
Based on the recognized value 1.759×10^{11} C/kg, measurement error is						0.8%

4) Observe the deflection of electrons in an electric field

   Turn the adjusting knob for excitation current counterclockwise all the way to the end and then set the current polarity switch to off position. Set the voltage polarity switch to positive at upper plate while monitoring electrons to be deflected upwards.

5) Post-experiment restoration of settings

   After the experiment, please make sure the settings of the front panel are restored as:

   Knob for adjusting acceleration voltage: turned all the way to end of counterclockwise

   Current polarity switch: off

   Knob for adjusting excitation current: turned all the way to end of counterclockwise

   Voltage polarity switch: off

   Knob for adjusting deflection voltage: turned all the way to end of counterclockwise

Note:

1) For better visibility of the electron beams, the experiment should be conducted in a dim room.
2) For higher accuracy measurements, please use voltmeter/ammeter (or digital

multimeters) to connect them to the rear panel of the apparatus for voltage and current measurements, respectively. The readings from the front panel are for coarse monitoring purpose only.

Caution:

1) *Restore the metal bar to the current terminals on the rear panel when an external multimeter is removed, as otherwise an open circuit would occur and no current would be supplied to the Helmholtz Coils (**Warning: never short cut the two "Voltage" terminals**).*

2) To prolong the lifetime of the Lorentz force tube, when the apparatus is idle, set all voltage and current to minimum.